**Bournemouth University**

# MATHEMATICAL SKILLS PRACTICE

# for

# ENGINEERS
# and
# DESIGNERS

## *Introduction*

### What is this booklet for?

Mathematics can be a difficult subject. We know. However, no matter which university you choose to study engineering or design, you will encounter mathematics. Now, as any runner, footballer or singer knows, it is necessary to warm up before performing. So, in the same way before you start your university course, you need to "warm up" by doing some mathematics. This booklet includes material on a number of mathematical topics that you have probably met previously, and which will be used frequently in some of the modules you will be studying. If you can use these techniques easily and reliably before you start your course, you will be better able to understand the new topics you meet, helping you to achieve greater success in your degree.

This booklet is designed to help you at an early stage of your University course to identify any areas you haven't covered, or aren't too sure about. If you aren't fluent in any of these topics, or if there are any you haven't met, firstly, **don't worry** and secondly, we'll help you improve in those topics during your first year. To help you, there are references to support material to be found on the web for most sections in this booklet - details of which are on the next page.

Of course, you don't *have* to use this booklet, but we hope you will find it useful and that it helps you to arrive at Bournemouth University ready to tackle a challenging and interesting degree!

### What happens when I arrive at University?

In the induction week you will take a diagnostic quiz on similar, GCSE level, material. ***This is now normal practice at most universities - although most don't tell you beforehand!*** Marks obtained for the quiz or for this booklet are not officially recorded. They are simply to help you identify any help you may need. Your tutor will discuss your quiz result with you and your success with this booklet - feel free to bring it along. If you've had problems then you'll be advised on the best method of help. This may include directed reading, the use of the university's Maths Centre, or a one-hour-per-week 'Maths Help' course through the first year. End of first year examination results over the past decade or more indicate that taking the 'Maths Help' course has proved particularly successful.

### Note for students without A-level or equivalent in Mathematics

Students entering our courses come with a wide background in mathematics so don't worry if you haven't seen some of the topics before (especially section 8 on calculus) - some of these topics will be taught in your first year at Bournemouth anyway - some you won't even see at all. However, the earlier sections, especially 1, 2, 4 and 5, are extremely important and you should try to complete as much as possible of these parts at least. Even if you *have* done A-level, this booklet will be useful for you to reassess your capabilities at this level.

### How should I use this booklet?

Here are some suggestions about how to best use the booklet:

1. Although you can use a copy of this workbook online, it is better to use a printed copy.
2. Do your working in the space provided so that all your working is contained within one document. If you run out of space, use extra paper (this is *not* going to be marked!).
3. *Do not use your calculator*. The arithmetic is straightforward. You can leave answers that appear as, say, $\sqrt{13}$ in the form $\sqrt{13}$.
4. Try at least a few exercises from most sections, even if you are already confident. This will help you to 'brush up' on material that you have not used for a while.
5. If you are not really confident on any section, then you should attempt *all* exercises. There is online help available, or perhaps find a textbook or past notes to help you reinforce these topics.

6. If you come across a section you cannot do, don't stop there. You will probably be able to go on to tackle questions from later sections.
7. Questions marked * have no answers given. We have given answers to most exercises, but not all. These are there to test your confidence, so check your working carefully for each of them!

**Is it assessed?**
This material will *not be directly assessed and will not count towards your coursework or examination marks*. Of course, some this material will necessarily be a prerequisite for, or be contained within, taught material in your normal course subjects.

**Mathematics Centre**
Bournemouth University's Mathematics Centre provides support for any student needing help in Mathematics. Appointments can be booked by individuals or by groups. Details can be found on the University web site, or ask your tutor for details.

**Useful material on the web**
In the following material, reference is made to relevant worksheets. The reference "[see W 4]", for example, indicates that the topic can be found in Worksheet 4. All worksheets, originally written for use by students at Hull University, can be found on
http://www.studyadvice.hull.ac.uk/Intranet/helpintranet/Mathsdownload.htm
and can be downloaded onto your computer and either read on the screen or printed out.

Other sites which you may find helpful are, for Algebra http://www.algebrahelp.com/, for Calculus http://math.about.com/cs/calculus/ and for a wide range of topics in mathematics http://www.searchbeat.com/Science/Math/ - there are, of course, many others!

**Acknowledgements**
This booklet has been adapted by Peter Edwards, Bournemouth University from material by Don Maskell (Mathematics Tutor, Study Advice Services, University of Hull), Dr John Appleby and other staff of the Faculty of Engineering of Newcastle University, based in turn on a similar booklet produced by Dr Tony Croft of Loughborough University.

This booklet is part of a project is funded by PROGRESS, an FDTL3 project on progression in Engineering. Our thanks go to Dr. S. Pulko, PROGRESS Project Supervisor, for permission to adapt this booklet for use at Bournemouth University.

***Contents***

**Which sections should I do?**

Of course, you don't ***have*** to study anything in this booklet, but to come well prepared for your course, it would be helpful if you studied the sections indicated in the table below. The following key is used to help prioritise your study:

| | |
|---|---|
| *** | highly recommended |
| ** | recommended |
| * | not absolutely necessary – covered in your course |
| X | not necessarily required for your course |

This booklet is intended for students coming to Bournemouth University mainly for the following courses.

1. **PD** Product Design (both BA and BSc)
2. **CAPD** Computer Aided Product Design
3. **DE** Design Engineering
4. **ECP** Electronics Combined Programme (four streams)
   - Computer Communications
   - Medical Electronics
   - Applied Computing & Electronics
   - Electronic Systems Design

The table below indicates the areas you should concentrate on for each course:

| Section | PD | CAPD | DE | ECP |
|---|---|---|---|---|
| 1 | *** | *** | *** | *** |
| 2.1 – 2.4 | *** | *** | *** | *** |
| 2.5 | X | X | ** | *** |
| 2.6 – 2.7 | ** | ** | ** | ** |
| 2.8 – 2.9 | X | X | * | * |
| 3 | ** | ** | ** | ** |
| 4 | *** | *** | *** | *** |
| 5 | *** | *** | *** | *** |
| 6.1 – 6.2 | ** | ** | * | * |
| 6.3 – 6.5 | * | * | ** | ** |
| 7.1 – 7.2 | *** | *** | *** | *** |
| 7.3 – 7.4 | * | * | ** | ** |
| 8 | X | X | * | * |

… and remember, if you have problems with any section, you can always check out the underlying theory by using the appropriate theory sheet from
***http://www.studyadvice.hull.ac.uk/Intranet/helpintranet/Mathsdownload.htm***

e.g. in Question 1, for example, it says [for sections 1.1 – 1.4 see W 23]. Go to the above web page and let the cursor hover over the "Download" links. They are in numerical order, so hovering over the next-to-last "Download" in the list will display the link to W23v3.pdf.
(Note: you will need the *Adobe Acrobat Reader* installed on your PC to view these files.
This is available for free download from http://www.adobe.com/products/acrobat/alternate.html)

***1. Numbers etc.*** [for sections 1.1 – 1.4 see W 23]

**1.1** Round each of the following to the number of decimal places given afterwards in brackets:

a) 3.1415924 (3) b) 3.1415924 (4) c) 1.4142 (2) d) 0.003299 (3)

e) 100.0423 (1) *f) 0.00409 (2) g) –34.567 (1) *h) 0.5499 (1)

**1.2** Round each of the following to the number of significant figures given afterwards in brackets:

a) 0.003299 (2) b) 100.0463 (5) c) 100.0463 (4) d) 1473.3 (2)

e) 14.548 (3) *f) –0.5557 (2) g) 0.0000034 (5) *h) 17001.3 (1)

**1.3** Put each number into scientific (standard) notation, e.g. $3.45\times10^{-3}$.

a) 34.56 b) 1089.4 c) 0.3027 d) 0.000552

e) –5.63 *f) –1001.1001 g) 0.0000004 *h) –99000.0

**1.4** Convert each number to ordinary decimal notation, e.g. 0.000403 :

a) $3.5\times10^{-3}$ b) $-2.071\times10^{5}$ c) $9.930\times10^{-1}$ d) $0.207\times10^{-2}$

e) $4.2156\times10^{4}$ *f) $3.14159\times10^{2}$ g) $1.00\times10^{6}$ *h) $-4\times10^{-7}$

**1.5** Put a < or > sign between each pair of numbers to show which is greater and which is less:

a) 9901 10032 b) –3 –7 c) 4 –17 d) –99.43 0.02

e) 0 –0.341 *f) $10^{2}$ $10^{5}$ g) $10^{-2}$ $10^{-3}$ *h) $-4\times10^{3}$ $3\times10^{4}$

**Working Space**

**Working Space**

**1.6** In each case, multiply the number by 100, also divide it by 1000: [see W23]

a) 3.1416 b) –453.001 c) 207.1 d) –0.00302

e) $3\times10^{4}$ *f) $7.5\times10^{2}$ g) $-3.2\times10^{-3}$ *h) $27.4\times10^{-2}$

**1.7** Write each decimal as a fraction, and each fraction as a decimal: [see W6]

a) 0.2 b) 0.125 c) –0.75 d) 0.375

e) $\frac{1}{4}$ *f) $-\frac{7}{5}$ g) $\frac{3}{40}$ *h) $\frac{1}{12}$

**1.8** Find the Highest Common Factor and Lowest Common Multiple of each pair or triple of numbers (e.g. 8 and 12 have HCF of 4 and LCM of 24):

a) 4 8 b) 6 9 c) 4 15 d) 6 8

e) 10 15 *f) 12 15 g) 5 12 *h) 30 40 55

**1.9** Calculate each of the following:

a) (–3) × (4), b) (–5) × (–7), c) – (–5) × (–4), d) (–1.5) × (–4), e) 7 - 3 × 4, *f) - 5 - 4 × 2

g) $4-\frac{3}{5}-\frac{1}{5}$, *h) 12 ÷ 4 ÷ 2 (what would your calculator give for this – think before trying it!)

**Working Space**

**Working Space**

## 2. Fractions, Indices and Logarithms

**2.1** Cancel each fraction to leave it in its simplest form (e.g. $\frac{18}{12}$ becomes $\frac{3}{2}$): [see W5]

a) $\frac{6}{4}$, b) $\frac{30}{18}$, c) $-\frac{12}{9}$, d) $\frac{36}{28}$

**2.2** Simplify each expression, leaving the result as a single improper fraction, cancelled down to its lowest terms (e.g. $\frac{11}{8}$ not $1\frac{3}{8}$ or $\frac{22}{16}$): [see W5]

a) $\frac{1}{2}+\frac{1}{3}$ b) $\frac{2}{3}+\frac{5}{6}$ c) $\frac{3}{4}-\frac{5}{6}$ *d) $\frac{1}{3}+\frac{3}{4}-\frac{1}{5}$

**2.3** Multiply or divide fractions, simplifying the result in each case: [see W5]

a) $\frac{2}{3}\times\frac{1}{4}$ b) $\frac{3}{7}\times\frac{49}{6}$ *c) $-\frac{3}{4}\times\frac{6}{5}$ d) $\left(-\frac{9}{5}\right)\times\left(-\frac{15}{8}\right)$

e) $\frac{2}{3}\div\frac{1}{6}$ f) $\frac{3}{4}\div\frac{2}{3}$ g) $-\frac{1}{5}\div\frac{3}{5}$ *h) $\frac{5}{12}\div\frac{10}{9}$

**2.4** Simplify each expression: [see W23]

a) $\left(\frac{x}{2}\right)\left(\frac{3x^2}{4}\right)$, b) $\left(\frac{x^2}{6}\right)\left(\frac{9}{x}\right)$, c) $\left(\frac{2x^2y}{5z}\right)\left(\frac{3yz^2}{4x^4}\right)$, d), $\left(\frac{1}{x^2}\right)\div\left(\frac{2}{x}\right)$, e), $\left(\frac{a}{a+1}\right)\div\left(\frac{3a^2}{a^2+1}\right)$,

*f) $\left(\frac{x}{2yz}\right)\div\left(\frac{x^2}{y(z+1)}\right)$, g) $\left(\frac{3b}{b+1}\right)\div\left(\frac{2b}{b^2-1}\right)$, *h) $\left(\frac{3x+6}{2y^3}\right)\div\left(\frac{6x}{3yz}\right)$

**Working Space**

**Working Space**

**2.5** Put over the lowest common denominator, and simplify the result, in each case. [see W22]

$$\left(\text{e.g.}\ \frac{1}{x}-\frac{3}{x^2}=\frac{x-3}{x^2};\qquad \frac{1}{x-2}-\frac{1}{x+3}=\frac{5}{(x-2)(x+3)}\right)$$

a) $\frac{3}{x-1}+\frac{2}{x+2}$, b) $\frac{4}{y+3}-\frac{3}{y-3}$, c) $\frac{1}{2x+3}+\frac{4}{x-2}$, d) $\frac{a}{y}-\frac{a+2}{2y-1}$

e) $\frac{3}{u}-\frac{2}{u+1}+\frac{4}{u-2}$, f) $\frac{1}{2}z+\frac{3}{z}-\frac{2}{z^2}$, g) $1+\frac{2x-3}{x^2+1}-\frac{3}{4x}$, *h) $2x-1+\frac{3}{2(x-2)}-\frac{1}{2(x+3)}$

**2.6** Give the value of each of the following: [see W1]

a) $2^{-3}$, b) $13^0$, *c) $4^{-1}$, d) $\left(\frac{1}{2}\right)^{-2}$, e) $8^{-\frac{1}{3}}$, f) $-16^{\frac{1}{4}}$, g) $27^{-\frac{2}{3}}$, *h), $32^{\frac{3}{5}}$ [W1/4]

**Working Space**

**Working Space**

**2.7** Simplify each of the following (e.g. $10^2 \times 10^3 = 10^5$): [see W1]

a) $10^3 \times 10^4$ b) $2^5 \times 2^{-3}$ c) $x^{0.5} x^{1.5}$ d) $x^5 / x^2$ e) $5^7 \div 5^3$

f) $y^{-\frac{1}{2}} y^{-\frac{5}{2}}$ g) $(2x)^3$ h) $\left(4x^2\right)^{\frac{1}{2}}$ i) $\left(x^2\right)^3$ *j) $\left(3y^{\frac{3}{2}}\right)^2$

k) $\left(z^{-1}\right)^3$ *l) $\left(x^{-1}\right)^{-1}$ m) $\left(9x^{-2}\right)^{\frac{1}{2}}$ *n) $\left(10^{-3.5}\right)^0$ o) $\left(\frac{1}{2}w^3\right)^{-1}$

**2.8** Give the value of each: [see W20]

a) $\log_{10}(100)$ b) $\log_{10}(1000000)$ c) $\log_{10}(0.1)$ d) $\log_{10}\left(\frac{1}{1000}\right)$

e) $\log_2(8)$ *f) $\log_2\left(\frac{1}{16}\right)$ g) $\log_e\left(e^{-3}\right)$ *h) $\log_e\left(\frac{1}{e}\right)$

**2.9** Give the result as the logarithm of a single number, or as a value where possible: [see W20]

a) $\log 3 + \log 4$ b) $\log 16 - \log 2$ c) $3\log 2 - 2\log 4$ d) $-\log\frac{1}{2}$

e) $10^{\log_{10} 5}$ *f) $-2e^{\ln 2}$ g) $\log_e 1$ *h) $e^{-\ln x}$

**Working Space**

**Working Space**

*Standard prefixes*: $m$ = milli = $10^{-3}$, $k$ = kilo = $10^{3}$, $M$ = mega = $10^{6}$, $\mu$ = micro = $10^{-6}$, (also $c$ = centi = $10^{-2}$).

*SI Units:* $m$ = metres, $s$ = seconds, $N$ = newtons, $kg$ = kilogrammes, $V$ = volts, $F$ = farads, $C$ = coulombs, $J$ = joules, $g$ =9.81 $m\,s^{-2} \cong 10\ m\,s^{-2}$ (also $l$ = litres).

**3.1** How many ..... ? (e.g. How many $mm$ in one $m$? Answer: $10^{3}$) [see W23]

a) $cm$ in one $m$ b) $ml$ in 2.5$l$ c) $s$ in 1$hr$ 30$mins$ d) $mV$ in 2$kV$

e) $m^2$ in 5$km^2$ *f) $\mu F$ in 2.3$F$ g) $\mu m$ in 50.3$mm$ *h) degrees in 2.5 revolutions

**3.2** Convert to the units given: [see W23]
a) $5m^2$ to $mm^2$, b) $3ms^{-1}$ to $km/hr$, c) $5Nmm^{-2}$ to $Nm^{-2}$, *d) $gmm^{-2}$ to $kgm^{-2}$

**3.3** Give the value in each case (with units): [see W23]
a) 3.6$m$ / 3$mm$, b) $10m^2$ / 2.5$m$, c) $4ms^{-1} \times 5mins$, *d) $\rho gh$, where $\rho = 3kgm^{-3}$, $h = 2m$

**3.4** A graph of pressure ($y$ axis) against 1/Volume ($x$ axis) goes through the points ($1/V = 5m^{-3}$, $p = 2000\ Nm^{-2}$), ($1/V = 15m^{-3}$, $p = 4000\ Nm^{-2}$). Find the gradient (with units).

**3.5** If the area of a rectangle is $3.6m^2$, and one side has length 180$mm$, what is the length of the other side in $m$?

*__3.6__ Water has density $1g / cm^3$. If steel has density 7.8 times that of water, what is the density of steel in tonnes per cubic metre (1 $t$ = 1000 $kg$)?

**Working Space**

**Working Space**

**4.1** Expand and collect terms: [see W2]
a) $2x(3-4x)$, b) $u-1+2(3-u)$, c) $2y+4(y-2)$, d) $z(z+2)-(z-2)$,

e) $3u(u+2)-2u(4-u)$, *f) $5x(2x-3)-1+2x-x(4-3x)$

**4.2** Expand and collect terms:[see W2]
a) $(x+1)(x+4)$, b) $(u-3)(u+4)$, c) $(y+3)(y-3)$, *d) $(2z-1)(2z+1)$,

e) $(x+2)(x+y-3)$, f) $(2x-y+3)(3x+2y-2)$, g) $(x+y)^2-(x-y)^2$,

h) $(a+b)^3$, *i) $(x+1)(x^2-x+1)$, j) $\left(x+\frac{1}{x}\right)^2$, k) $\left(\frac{1}{x}-\frac{2}{x^2}\right)^2$

**Working Space**

**Working Space**

**4.3** Factorise as far as possible in each case (e.g. $18x^2y + 8xy^2 = 2xy(9x + 4y)$ ): [see W2]

a) $6x + 3xy$, b) $8u^2 - 6uv$, c) $\frac{1}{2}x^3 - \frac{1}{4}x^2$, *d) $4ab^2c - 9ac^3$,

**4.4** Factorise into two brackets:[see W2]

a) $x^2 + 3x + 2$, b) $y^2 - 4y + 3$, c) $v^2 - 2v - 15$, *d) $z^2 + z - 20$, e) $2x^2 + 3x + 1$,

*f) $2u^2 + u - 3$, g) $6z^2 - 13z + 6$, h) $x^2 + 6x + 9$, i) $a^2 - b^2$, j) $4x^2 - 9y^2$,

k) $x^2 - 3$, l) $u^2 - 4u + 4$

**Working Space**

**Working Space**

***5. Formulae and equations***

**5.1** Solve each equation:[see W2]

a) $2x-3=4-3x$, b) $4y+1=\frac{1}{2}y-\frac{3}{2}$, c) $\frac{3-z}{2}=\frac{1-3z}{4}$, d) $\frac{1}{u}=\frac{3}{u-1}$,

e) $\frac{2x-1}{3x+1}=\frac{4-2x}{1-3x}$, * f) $\frac{z+1}{z-3}=\frac{4-z}{2-z}$

**5.2** Solve each quadratic equation *by factorising*: [see W4]

a) $x^2-3x+2=0$, b) $y^2-y-12=0$, * c) $z^2+2z-24=0$, d) $2u^2-u-1=0$,

e) $v-3v^2+2=0$, f) $4w^2+3w-10=0$, g) $x^2=4x$, * h) $-3y=-5y^2$

**Working Space**

**Working Space**

**5.3** Transpose each formula to make the bracketed variable the subject: [see W3]

a) $x = 3t - 2$ $(t)$, b) $y = \dfrac{3}{2 - x}$ $(x)$, c) $T = 2\pi\sqrt{\dfrac{L}{g}}$ $(L)$, d) $u = \dfrac{3v - 2}{2v + 1}$ $(v)$,

e) $Q = \dfrac{\pi a^4 p}{8\mu L}$ $(p)$, f) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ $(R)$, *g) $pV^{\gamma} = C$ $(\gamma)$

**5.4** Solve the simultaneous equations: [W4]

a) $2x - 3y = 4$ b) $4x + 3y = 0$ c) $2u + v = 3$ *d) $x^2 - y = 0$

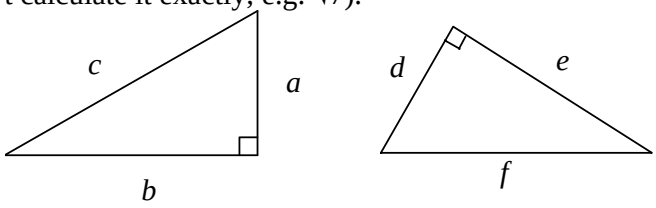
$-x + 2y = 5$, $3x - 5y = 2$, $\frac{1}{2}u + \frac{3}{2}v = 1$, $2x + y = -1$

**Working Space**

**Working Space**

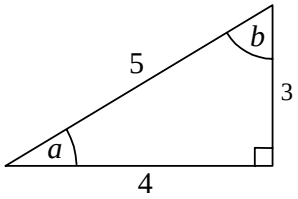
***6. Trigonometry etc.***

**6.1** In each case, given two sides, find the third side (your answer may be left as a square root if you can't calculate it exactly, e.g. √7).




a) $a = 3$, $b = 5$, find $c$, b) $a = 4$, $c = 7$, find $b$, c) $d = 5$, $e = 12$, find $f$, d) $e = 6$, $f = 7$, find $d$.

**6.2** For the triangle shown, write down $\sin a$, $\cos a$, $\tan a$, $\sin b$, $\operatorname{cosec} a$, $\cot b$, $\sec a$




**6.3** Give the values of $\sin 30°$, $\sin 45°$, $\sin 60°$, $\cos 0°$, $\tan 45°$, $\cos 30°$, $\sin 90°$, $\sin 0°$, $\cos 90°$ *without using a calculator*, and find the value of $(\sin\theta)^2 + (\cos\theta)^2$ for several values of $\theta$.

**6.4** Convert between degrees and radians:
a) 30º, b) 90º, *c) 45º, d) 150º, e) 270º, f) 315º, g) π/2, *h) π/4, i) 2π/3, j) 5π/4, *k) 9π/4

**6.5** Give values of
$\tan 60°$, $\sin 120°$, $\sin(3\pi/4)$, $\sin 210°$, $\cos(5\pi/6)$, $\tan 215°$, $\cos(3\pi/2)$, $\sin(-30°)$

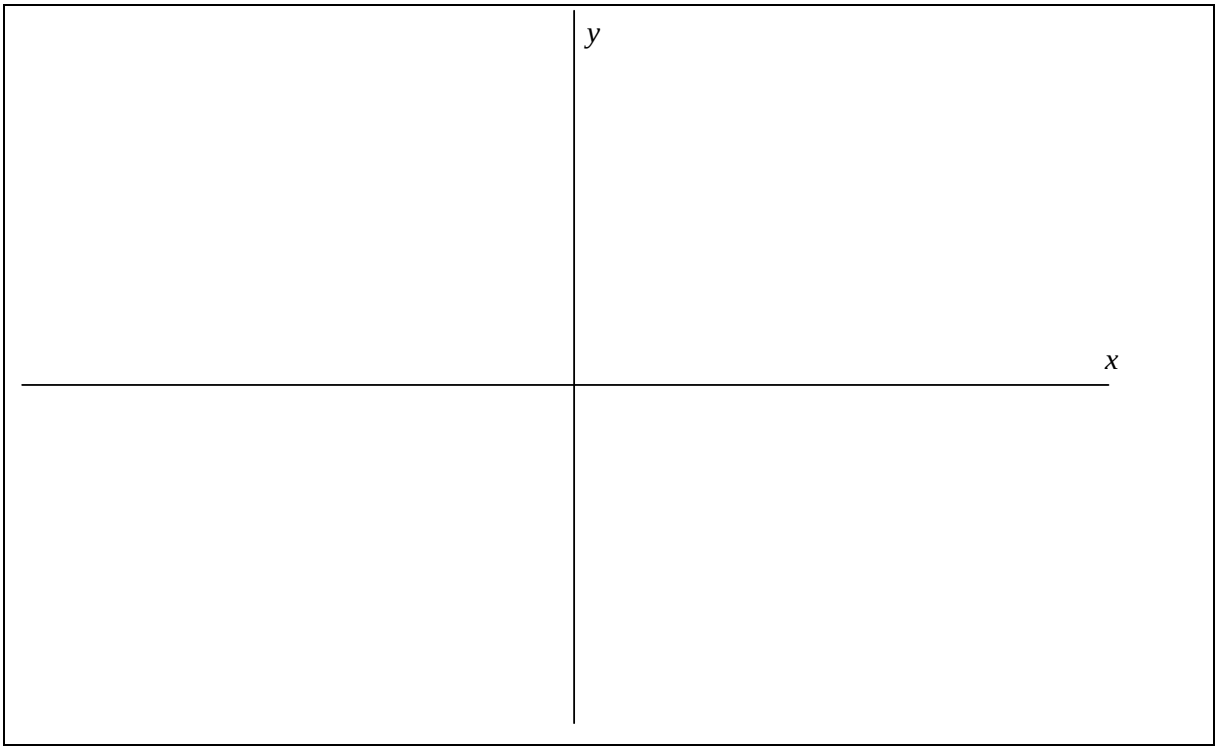
**Working Space**

**Working Space**

***7. Graphs***

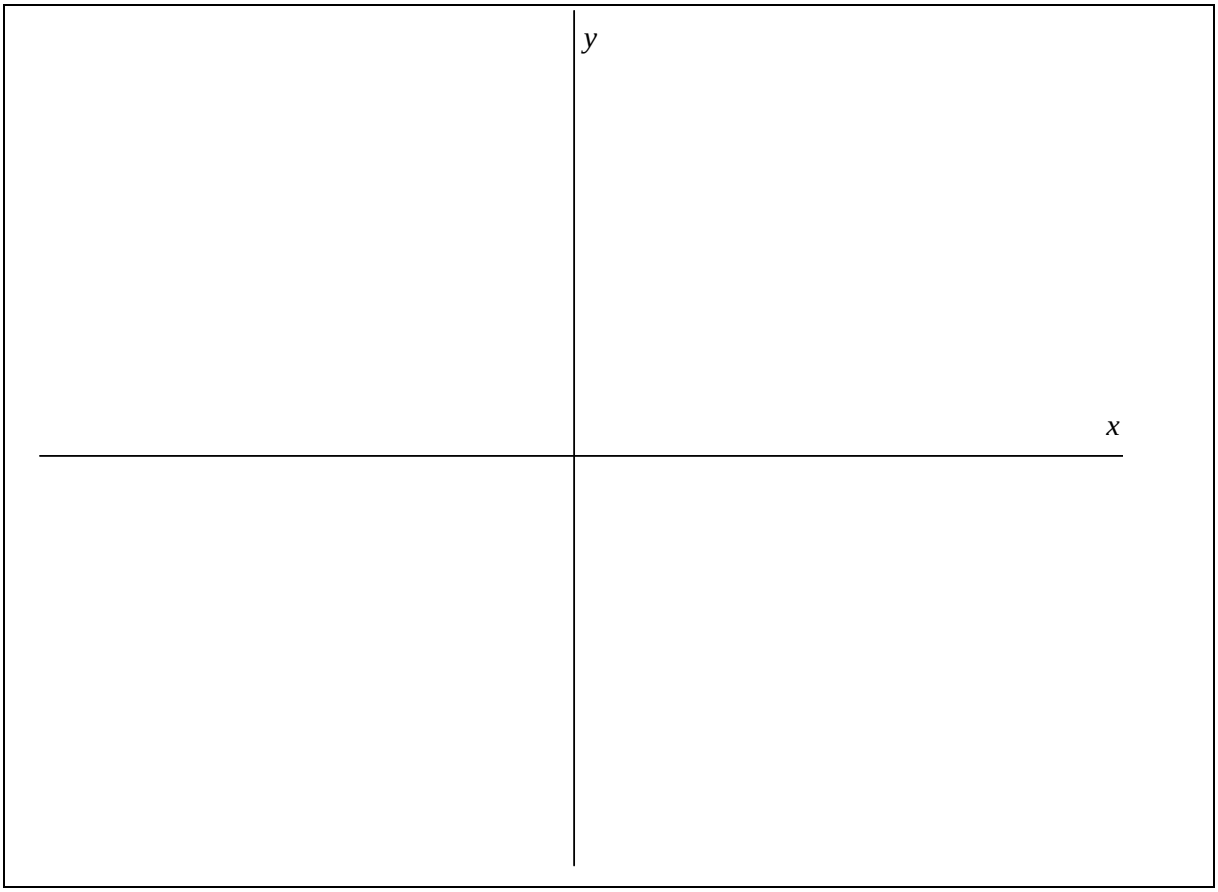
**7.1** Find the equation of the straight line in each case (use variables *x, y*):
a) through (0, 0) and (3, 6), b) through (-1, 3) and (1, 1), c) through (-3, -4) and (-5, -1),
d) through (2,-3) with gradient $\frac{1}{2}$, e) through (1, 3), perpendicular to the line $y = 1 - 2x$

**7.2** Sketch the graphs (all on one diagram) of: [see W17]
a) $y = 2x - 1$, b) $2x + 4y - 1 = 0$, c) $x = 3$, d) $y = -2$, and give the gradient for case b).

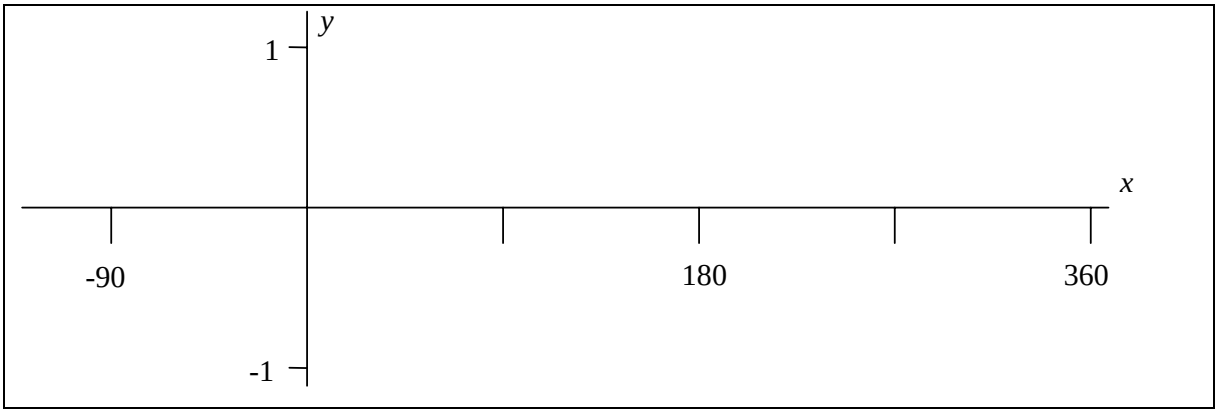


**7.3** Sketch (on one diagram) the graphs of [see W17]

a) $y = x^2 - 1$, b) $y = 4 - x^2$, c) $y = -\frac{1}{x}$, d) $y = 2 - \frac{3}{x}$, e) $y^2 = 4x$




**7.4** Sketch the graphs of $y = \sin x$, $y = \cos x$, $y = \sin 2x$ for values of $x$ from -90º to 360º.



***8. Differentiation and simple integration***

**8.1** Differentiate each expression (note *a* is a constant): [see W8]

a) $y = x^2 - 1$, b) $y = \frac{1}{2}x^3 - 3 + 2ax$, c) $z = -\frac{1}{u}$, d) $v = \frac{u - 3u^2}{4a}$, e) $z = \frac{4}{3u^2} - \frac{\sqrt{u}}{5}$

**8.2** Differentiate each expression (*a* is constant): [see W9]

a) $y = \cos x$, b) $v = e^{2u}$, c) $f = a\sin 3\theta$, d) $s = 2\ln t - 3e^{-t} - 3a$, e) $y = 2a\pi \tan x - \frac{a-1}{x}$

**Working Space**

**Working Space**

**8.3** Differentiate: [see W8]

a) $y = 3x^2 \sin x$, b) $z = \dfrac{2-3w}{1+4w}$, c) $s = \cos\left(t^2\right)$, d) $w = \dfrac{3m - 2\sqrt{m}}{m^3}$

**8.4** Give the indefinite integral of: [W10]

a) $x^3 - 3x + 1$, b) $-\cos x + e^{-x} + \dfrac{3}{x}$, c) $4\sin 2x - \frac{1}{2}\sec^2 x$, d) $\dfrac{t\sqrt{t} - 3}{t^2}$

**Working Space**

**Working Space**

***Answers to Exercises***

Exercises marked (*) have *no* answers given here. You must check these carefully yourself! Also note that many answers have *alternative forms*. For example, whether or not you expand brackets, factorise or put fractions over a common denominator is often a matter of choice. So, if your answer is in a slightly different form, but which is equivalent, that's acceptable.

1.1 a) 3.142, b) 3.1416, c) 1.41, d) 0.003, e) 100.0, f) *, g) -34.6, h) *

1.2 a) 0.0033, b) 100.05, c) 100.0, d) 1500, e) 14.5, f) *, g) 0.0000034000, h) *

1.3 a) $3.456 \times 10^{1}$, b) $1.0894 \times 10^{3}$, c) $3.027 \times 10^{-1}$, d) $5.52 \times 10^{-4}$, e) $-5.63 \times 10^{0}$, f) *, g) $4 \times 10^{-7}$, h) *

1.4 a) 0.0035, b) -207100, c) 0.9930, d) 0.00207, e) 42156, f) *, g) 1000000, h) *

1.5 a) <, b) >, c) >, d) <, e) >, f) *, g) >, h) *

1.6 a) 314.16, 0.0031416, b) -45300.1, -0.453001, c) 20710, 0.2071, d) -0.302, -0.00000302, e) $3 \times 10^{6}$, 30, f) *, g) $-3.2 \times 10^{-1}$, $-3.2 \times 10^{-6}$, h) *

1.7 a) $\frac{1}{5}$, b) $\frac{1}{8}$, c) $-\frac{3}{4}$, d) $\frac{3}{8}$, e) 0.25, f) *, g) 0.075, h) *

1.8 a) 4, 8, b) 3, 18, c) 1, 60, d) 2, 24, e) 5, 30, f) *, g) 1, 60, h) *

1.9 a) -12, b) 35, c) -20, d) 6, e) -5, f) *, g) $\frac{16}{5}$, h) *

2.1 a) $\frac{3}{2}$, b) $\frac{5}{3}$, c) $-\frac{4}{3}$, d) $\frac{9}{7}$

2.2 a) $\frac{5}{6}$, b) $\frac{3}{2}$, c) $-\frac{1}{12}$, d) *

2.3 a) $\frac{1}{6}$, b) $\frac{7}{2}$, c) *, d) $\frac{27}{8}$, e) 4, f) $\frac{9}{8}$, g) $-\frac{1}{3}$, h) *

2.4 a) $\frac{3x^3}{8}$, b) $\frac{3x}{2}$, c) $\frac{3y^2z}{10x^2}$, d) $\frac{1}{2x}$, e) $\frac{a^2+1}{3a(a+1)}$, f) *, g) $\frac{3(b-1)}{2}$, h) *

2.5 a) $\frac{5x+4}{(x+2)(x-1)}$, b) $\frac{y-21}{y^2-9}$, c) $\frac{9x+10}{(2x+3)(x-2)}$, d) $\frac{ay-2y-a}{y(2y-1)}$, e) $\frac{5u^2+5u-6}{u(u+1)(u-2)}$, f) $\frac{z^3+6z-4}{2z^2}$, g) $\frac{4x^3+5x^2-8x-3}{4x(x^2+1)}$, h) *

2.6 a) $\frac{1}{8}$, b) 1, c) *, d) 4, e) $\frac{1}{2}$, f) -2, g) $\frac{1}{9}$, h) *

2.7 a) $10^{7}$, b) $2^{2}$, c) $x^{2}$, d) $x^{3}$, e) $5^{4}$, f) $y^{-3}$, g) $8x^{3}$, h) $2x$, i) $x^{6}$, j) *, k) $z^{-3}$, l) *, m) $3x^{-1}$, n) *, o) $2w^{-3}$, p) *

2.8 a) 2, b) 6, c) -1, d) -3, e) 3, f) *, g) -3, h) *

2.9 a) log 12, b) log 8, c) log $\frac{1}{2}$ = -log 2, d) log 2, e) 5, f) *, g) 0, h) *

3.1 a) 100, b) 2500, c) 5400, d) $2 \times 10^{6}$, e) $5 \times 10^{6}$, f) *, g) 50300, h) *

3.2 a) $5 \times 10^{6}$ mm$^2$, b) 10.8 km/hr, c) $5 \times 10^{6}$N.m$^{-2}$, d) *

3.3 a) 1200, b) 4 m, c) 1200 m, d) *

3.4 200 N.m 3.5 20 m 3.6 *

4.1 a) $6x-8x^2$, b) $5-u$, c) $6y-8$, d) $z^2+z+2$, e) $5u^2-2u$, f) *

4.2 a) $x^2+5x+4$, b) $u^2+u-12$, c) $y^2-9$, d) *, e) $x^2+xy+2y-x-6$, f) $6x^2+xy-2y^2+5x+8y-6$, g) $4xy$, h) $a^3+3a^2b+3ab^2+b^3$, i) *, j) $x^2+2+\frac{1}{x^2}$, k) $\frac{1}{x^2}-\frac{4}{x^3}+\frac{4}{x^4}=\frac{x^2-4x+4}{x^4}$

4.3 a) $3x(2+y)$, b) $2u(4u-3v)$, c) $\frac{1}{4}x^2(2x-1)$, d) *

4.4 a) $(x+2)(x+1)$, b) $(y-3)(y-1)$, c) $(v-5)(v+3)$, d) *, e) $(2x+1)(x+1)$, f) *, g) $(3z-2)(2z-3)$, h) $(x+3)^2$, i) $(a+b)(a-b)$, j) $(2x+3y)(2x-3y)$, k) $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$, l) $(u-2)^2$

5.1 a) $x = \frac{7}{5}$, b) $y = -\frac{5}{7}$, c) $z$ = -5, d) $u = -\frac{1}{2}$, e) $x$ = -1, f) *

5.2 a) $(x-2)(x-1)=0$ so $x$ = 1 or 2, b) $y$ = 4, -3, c) *, d) $u = 1, -\frac{1}{2}$, e) $v = 1, -\frac{2}{3}$, f) $w = \frac{5}{4}$, -2, g) $x$ = 0, 4, h) *

5.3 a) $t = \dfrac{x+2}{3}$, b) $x = 2 - \dfrac{3}{y}$ or $x = \dfrac{2y-3}{y}$, c) $L = \dfrac{gT^2}{4\pi^2}$, d) $v = \dfrac{2+u}{3-2u}$,

e) $p = \dfrac{8Q\mu L}{\pi a^4}$, f) $R = \dfrac{R_1 R_2}{R_1 + R_2}$, g) * (use logs)

5.4 a) $x$ = 23, $y$ =14, b) $x = \frac{6}{29}$, $y = -\frac{8}{29}$, c) $u = \frac{7}{5}$, $v = \frac{1}{5}$, d) *

6.1 a) $c = \sqrt{34}$, b) $b = \sqrt{33}$, c) $f$ = 13, d) $\sqrt{13}$

6.2 $\sin a = \frac{3}{5}$, $\cos a = \frac{4}{5}$, $\tan a = \frac{3}{4}$, $\sin b = \frac{4}{5}$, $\operatorname{cosec} a = \frac{5}{3}$, $\cot b = \frac{3}{4}$, $\sec a = \frac{5}{4}$

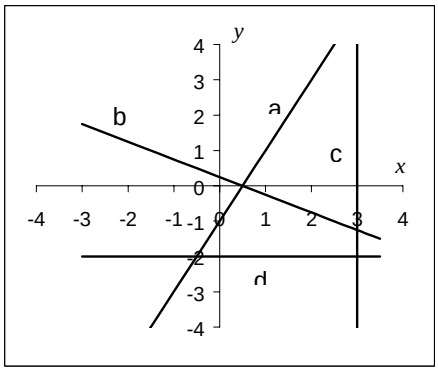
6.3 $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1, 1, \frac{\sqrt{3}}{2}, 1, 0, 0$. Also, note that $(\sin\theta)^2 + (\cos\theta)^2 = 1$ for *all* values of $\theta$.

6.4 a) $\frac{\pi}{6}$, b) $\frac{\pi}{2}$, c) *, d) $\frac{5\pi}{6}$, e) $\frac{3\pi}{2}$, f) $\frac{7\pi}{4}$, g) 90°, h) *, i) 120°, j) 225°, k) *

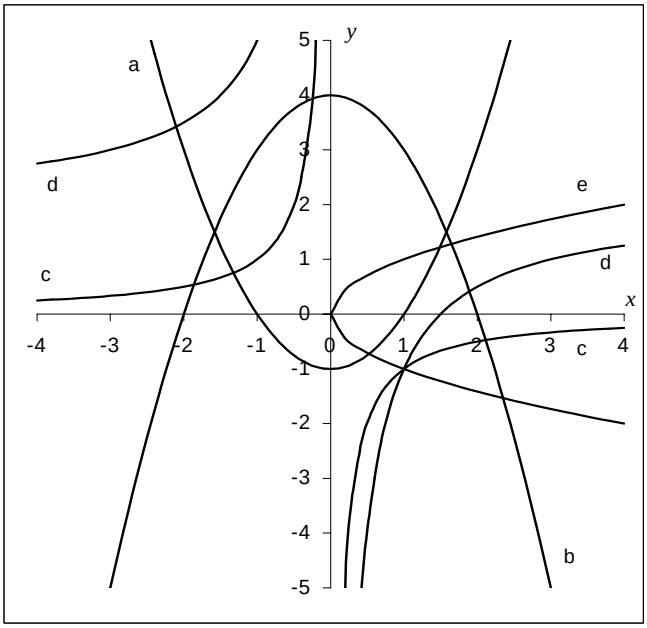
6.5 $\sqrt{3}$, $\frac{\sqrt{3}}{2}$, $\frac{1}{\sqrt{2}}$, $-\frac{1}{2}$, $-\frac{\sqrt{3}}{2}$, 1, 0, $-\frac{1}{2}$

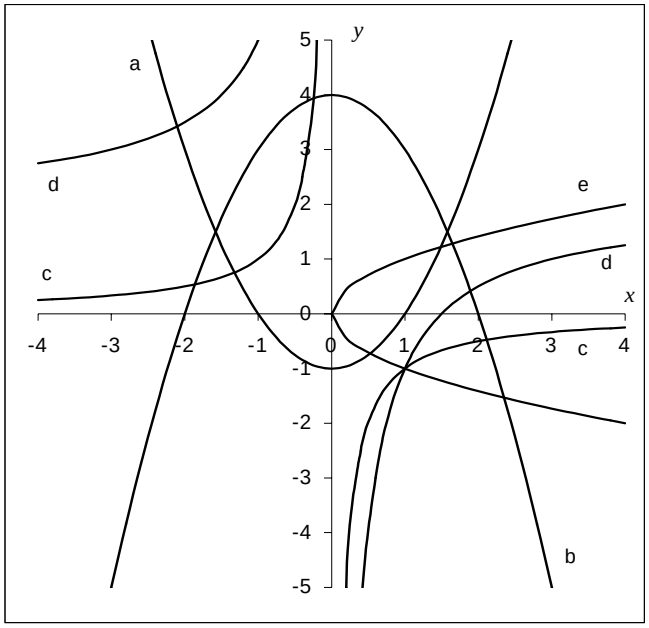
7.1 a) $y = 2x$, b) $y = 2 - x$, c) $y = -\frac{3}{2}x - \frac{17}{2}$, d) $y = \frac{1}{2}x - 4$, e) $y = \frac{1}{2}x + \frac{5}{2}$

7.2 Case b), gradient is - 0.5

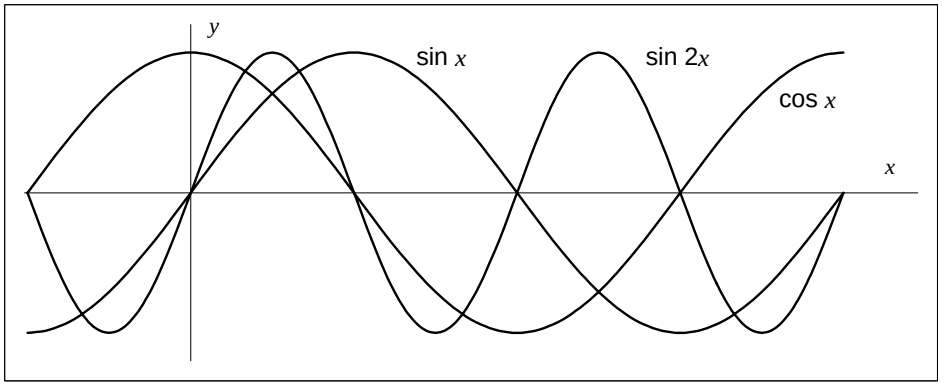



7.3






7.4




8.1 a) $\frac{dy}{dx} = 2x$, b) $\frac{dy}{dx} = \frac{3}{2}x^2 + 2a$, c) $\frac{dz}{du} = \frac{1}{u^2}$, d) $\frac{dv}{du} = \frac{1-6u}{4a}$,

e) $\frac{dz}{du} = -\frac{8}{3u^3} - \frac{1}{10\sqrt{u}}$

8.2 a) $\frac{dy}{dx} = -\sin x$, b) $\frac{dv}{du} = 2e^{2u}$, c) $\frac{df}{d\theta} = 3a\cos 3\theta$, d) $\frac{ds}{dt} = \frac{2}{t} + 3e^{-t}$,

e) $\frac{dy}{dx} = 2a\pi \sec^2 x + \frac{a-1}{x^2}$

8.3 a) $\frac{dy}{dx} = 6x\sin x + 3x^2\cos x$, b) $\frac{dz}{dw} = \frac{-11}{(1+4w)^2}$, c) $\frac{ds}{dt} = -2t\sin\left(t^2\right)$,

d) $\frac{dw}{dm} = -\frac{6}{m^3} + \frac{5}{m^{7/2}} = -\frac{6}{m^3} + \frac{5}{m^3\sqrt{m}} = \frac{5-6\sqrt{m}}{m^3\sqrt{m}}$

8.4 a) $\frac{1}{4}x^4 - \frac{3}{2}x^2 + x + C$, b) $-\sin x - e^{-x} + 3\ln x + C$, c) $-2\cos 2x - \frac{1}{2}\tan x + C$,

d) $2\sqrt{t} + \frac{3}{t} + C$