Probability of Undetected Errors of Optical Wireless Links.

Anthony C. Boucouvalas
Bournemouth University, Department of Electronics,
Fern Barrow, Poole, Dorset, BH12 5BB, U.K.
e-mail: tboucouv@bournemouth.ac.uk

ABSTRACT

This paper describes an analysis for the probability of undetected errors for SDLC/Manchester Encoded links. Results of mean time between undetected errors versus probability of line error rate are given. Such graphs are of interest to free space IR link designers offering better insight into the robustness of their design, for 16 or 32 bit CRCs. Coupled with the cost constraints it allows one to decide, with increased confidence, on the suitability of the combination of modulation scheme, framing and choice of CRC for the product under consideration. Furthermore the analysis offers a means for determining the weakest part (most susceptible to noise) of the SDLC framing.

Keywords: Optical wireless links, free space optical links, undetected errors, SDLC.

1. INTRODUCTION

When designing Free Space Optical Wireless Links, (IR Links), one of the many decisions which have to be taken is the choice of modulation scheme, the framing of the data, and the size of the CRC following the data within the frame. It is well known that the IR links are susceptible to interference from external noise sources, with result that the probability of error rate varies widely. A commonly used measure to determine the quality of the IR links is the probability of undetected errors. That is errors which have escaped the frame check sequence polynomial, determined by the CRC and line code. Such errors can propagate and damage the user application if not detected. The mean time between undetected frame errors, MTUE, [1], should be maximised. The choice of CRC therefore reflects on the quality of the IR link, and its cost.
Since IBM developed the Synchronous Data Link Control protocol, SDLC, it has been widely used and ISO modified it to become HDLC, High Data Link Control. CCITT then modified HDLC for its LAP (Link Access Procedure) as part of the X.25 network interface standard. The obvious advantage of using such framing is that there is wide availability of chips supporting such protocols, [2]. There are numerous suppliers of Serial Communication Controllers, (SCCs), supporting SDLC. This implies lower implementation costs. SCC chips provide a number of line coding schemes for the user, including Manchester encoding. Manchester has a lot of advantages for wireless links, including balanced code set, ease of clock extraction. and has been considered a candidate for wireless links. The purpose of this work is to calculate the MTUE for Manchester encoded data transmitted in SDLC format.

2. SDLC FRAMING.

The SDLC protocol is bit oriented and uses bit stuffing for data transparency. All bit oriented protocols use bit stuffing. Figure 1 shows the frame format of SDLC. It consists of a Begin of Frame flag, (BOF), Address Field, Control Field, Data, CRC and the End of Frame flag. The BOF for SDLC is 01111110. It is only one of two possible combinations that have six consecutive ones in SDLC. The other possibility is an abort character which consists of eight or more consecutive ones. This is because SDLC uses a process called bit stuffing.

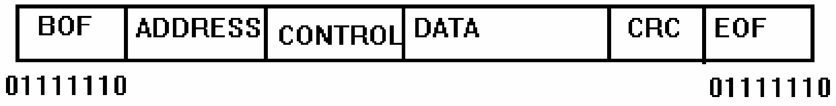

Figure 1. Frame format of SDLC protocol.

Bit stuffing is the insertion of a 0 as the next bit every time a sequence of five consecutive 1s is detected. On receive, the 0s are automatically removed after every consecutive group of five 1s. The removal of the 0 bit is called bit stripping.

The BOF signals the start of a frame and is limited to 8 bits. It is a unique combination of bits not encountered in the data field, due to the bit stuffing process. The BOF is also used for clock synchronisation and as a reference point for determining the position of the address and control fields.

The ADDRESS field is used to identify uniquely which stations the message it is intended for. Each station must have a unique address. It is 8 bit long, expandable to 32. For broadcast the address field is set to 11111111. The CONTROL field is used for initialisation of the system, frame sequence, to identify if the message is complete, error reporting etc.

The DATA field can be arbitrary long, containing the information data, but must be an integer multiple of 8 bits. It is identified by the BOF proceeding it and by the CRC following it.

The CRC is an error checking sequence: For SDLC a 16 bit CRC is normally used. The 32 bit CRC is normally used for CSMA/CD applications.

EOF is a one byte sequence identical to the BOF, and indicates when the transmission is complete.

3. ASSUMPTIONS.

The following assumptions are made for the analysis presented here:

a. Noise events are independent. The occurrence of one noise event does not change the probability of occurrence of the next noise event. This simplifies the analysis considerably.

b: Noise events are non-bursty. Each noise event affects the signal during one code bit. Unlike FDDI this does not result in bursty errors in the data bits.

c: The link is “Binary Symmetric” channel. This means that the probability of “high” being in error is the same as the probability of “low” being in error.

d: Noise events do not add or delete code bits. Only errors in interpreting them are modelled.

e: All data bit patterns are equally likely. For a worst case analysis it may be possible to use frames which are more likely to result in undetected errors. This has not been investigated.

f: Data bit errors in MAC layer components are not modelled.

g: For the calculations, unless otherwise stated, we assume a large frame size, F=8544 Manchester line code bits.

h: It is not easy to calculate the bounds on the damage caused by undetected errors, hence it is desirable to minimise the number of undetected errors per year. If a manufacturer produces one million devices per year, it may be desirable to reduce the number of undetected errors to 10^{-6}, resulting to a probability of about one undetected error per year, having a certain probability of resulting in a liability suit.

4. ERROR CALCULATIONS.

4.1. Effect of one or two line code noise events

It is necessary to determine the impact of noise on the coded data in more detail before we start the calculations on the probability of undetected errors.

We consider first a single noise event in the encoded data. Since we are considering Manchester Encoding, a single line code bit error would always be detected as code violation by the Manchester Decoder.

Therefore no single line code error should propagate, and it would be picked up by the decoder, before the CRC.
A single line error therefore does not cause any valid data and it is always detected.
Let us now consider the case when two line errors are present. If the two errors have at least one bit separation, then they would still cause individual code bit violations and be detected by the decoder, before the CRC.
However, two successive line code bit errors would not always cause a code violation. A double line code error causing a valid line code occurs when the double error occurs on the two line bits corresponding to a transition in the middle of the data bit. Sometimes there is also a transition at the edge between the two data bits. A double error then would result in a line code violation since it is equivalent to two single line bit errors in successive bits causing two violations and would be detected.
The probability of two successive errors in line bits causing an undetected data bit by the line code violations, in a frame of length F, is given by $(\frac{F-1}{2})p^2(1-p)^{F-2}$. Such a double line code error, causing a single data bit error would always be detected by the CRC. Therefore, two line code bit errors should not pose a problem. It is also assumed that two data bit errors will always be detected by the CRC. Some three data bit errors though will not.

4.2 Noise in BOF field.

The probability of undetected errors due to false BOF can be calculated as follows:
1. Probability of a single error in BOF, P_1, (double line code error in Manchester code, it can be easily shown that the probability of two, or more errors is far smaller, so it is ignored here).

P_1 = Probability of a double line bit error x probability of the remaining bits not being in error x the number of possible positions of error.

$$P_1 = (\frac{F-1}{2})p^2[1-p]^{16-2} \times \binom{8}{1}$$

2. P_2 = The probability that a BOF byte will be formed (due to errors) in F.
Within the frame field F we are interested for a sequence of data which due to single data bit error, (two successive errors in line bits), will result in a valid BOF. The newly formed BOF should also reside in an octet position with the final EOF sequence.
Therefore we are seeking the probability for a sequence of 8 bits of bit stuffed data which when hit by a single data bit error will result in a 01111110 sequence. The following six 8 bit sequences out of the possible $2^6 + 6$ =70, result in a BOF.

x
1. 0 1 1 1 1 1 0 0

x
2. 0 1 1 1 1 0 1 0

x
3. 0 1 1 1 0 1 1 0 x = position of data error

x
4. 0 1 1 0 1 1 1 0

x
5. 0 1 0 1 1 1 1 0

x
6. 0 0 1 1 1 1 1 0

P_2 = probability of a double error of no Manchester code violation, $\frac{p^2}{2}(F-1)$, x Probability of the remaining data bits not being in error, $(1-p)^{16-2}$ x possible error positions out of 70, $\frac{6}{2^6+6}$, and also finding a BOF on an octet within the frame of length F.

If we let $c=\frac{6}{2^6+6}(\frac{p^2}{2})(F-1)(1-p)^{14}$ then 1-c is the probability of 8 bits not being a BOF byte.

Hence $(1-c)^{\frac{F-4x16}{2x16}}$ is the probability that BOF is not present in any of the (F-4 bytes = F-4x16) bits of the SDLC frame, (4 bytes, consisting of CRC, Address and Control bytes). F being the field of the frame between BOF and EOF. The 2x16 factor in the denominator is due to Manchester code violation positions, (2) and due to the octet alignment (16), i.e. only half of those positions would be valid, since the other half would give code violations, and only 1/16 of those are octet aligned.
The probability that a BOF is present in the data is therefore:

$$1-(1-c)^{\frac{F-64}{32}}. \quad \text{Hence } P_2 = 1-\left[1-\frac{6}{70}(\frac{p^2}{2})(F-1)(1-p)^{14}\right]^{\frac{F-64}{32}}$$

3. P_3 = The probability that the following CRC is valid is 2^{-16}.

Hence
P(Undetected Errors due to error in BOF)= $P_1 x P_2 x P_3$

$$P(UEBOF) = 8(\frac{p^2}{2})(F-1)[1-p]^{14} \times \left[1-\left\{1-\frac{6}{70}(\frac{p^2}{2})(F-1)(1-p)^{14}\right\}^{\frac{F-64}{32}}\right] x 2^{-16}$$

Using
$(1+x)^n = 1+nx+\dots$ then for small p,

$$P(UEBOF) = 8(\frac{p^2}{2})(F-1)[1-p]^{14} \times \frac{F-64}{32} x \frac{6}{70}(\frac{p^2}{2})(F-1)(1-p)^{14} x 2^{-16}$$

or

$$P(UEBOF) = \frac{48}{70} x (\frac{p^4}{4})(F-1)^2 [1-p]^{28} \times \frac{F-64}{32} x 2^{-16}.$$

For p=2.*10e-10, F=8544, and for 2Mbit/s line rate for example, then P(UEBOF)=80.9*10e-36, or Mean time between undetected errors due to BOF is 1.67*10e24 years.

4.3 Noise in Data within the SDLC field F

Let us assume that some three data errors may go undetected by the CRC. One and two data bit errors are detected.

The errors may occur anywhere in the data, (octet alignment is not necessary).
The probability of three errors being undetected, using similar arguments as above can therefore be approximated as follows:

$$P(3UE) = \begin{pmatrix} \frac{F}{2} \\ 3 \end{pmatrix} \left(p^2 (F-1)/2\right)^3 (1-p)^{F-6} x 2^{-16}$$

For p=2.E-10, and F=8544, P(3UE)=2.618*10e-40.
This corresponds to a mean time between errors of MT(3UE)=5.173*10e29 years.

For the case of four undetected errors in F,

$$P(4UE) = \begin{pmatrix} \frac{F}{2} \\ 4 \end{pmatrix} \left(p^2 (F-1)/2\right)^4 (1-p)^{F-8} x 2^{-16}$$

For the same values of p and F as above, for a 2 Mbit/s line rate system P(4UE)= 1.802*10e-55, or MT(4UE)=7.516*10e44 a rare event.

4.4 SDLC EOF Undetected Errors.

The EOF byte in the SDLC protocol is the same as the BOF byte, 01111110. To calculate the probability of undetected error due to an error in EOF can be calculated as follows:

P(A double line code error in F, resulting in valid data) = $\frac{p^2}{2}(F-1)$.

P(Valid EOF byte arising from an error in eight bit sequence) = $\frac{6}{70}$

The packet must be octet aligned, so there are (F-4x16)/(2x16) possible positions.

The probability of undetected errors from the 16 bit CRC preceding the EOF is 2^{-16}.

Hence for 16 bit CRC $P(UE\, due\, to\, EOF) = \frac{F-4x16}{2x16} x \frac{6}{70} x \left(\frac{p^2}{2}\right)(F-1)(1-p)^{14} x 2^{-16}$

For F= 8544 line bits and p=2E-10, for a 2Mbit/s line rate system, P(UE due to EO)= 5.92*10e-20 which corresponds to MT(UE EOF)= 2.28*10e9 years.

This formula assumes that the violations have not been detected already by the CRC before we reach to the EOF.
A correction factor such as (1-P(UEBOF))(1-P(3UE))(1-P(4UE)) must be multiplied to the previous formula in order to account for the violations that have been detected already by the CRC before we reach to the EOF, due to BOF, 3 or 4 undetected errors..

This result indicates that undetected errors due to false EOF are limiting SDLC packet transmission integrity.
For 32 bit CRC we have $P(UE\, due\, to\, EOF) = \frac{F-4x16}{2x16} x \frac{6}{70} x \left(\frac{p^2}{2}\right)(F-1)(1-p)^{14} x 2^{-32}$,
again, multiplied by the correction factor (1-P(UEBOF))(1-P(3UE))(1-P(4UE))

Comparison using 16 and 32 bit CRC error protection is shown in Figure 2, where the line probability of error is plotted against the mean time between undetected errors in years due to the combined effects of BOF, 3 or 4 undetected errors, and EOF.

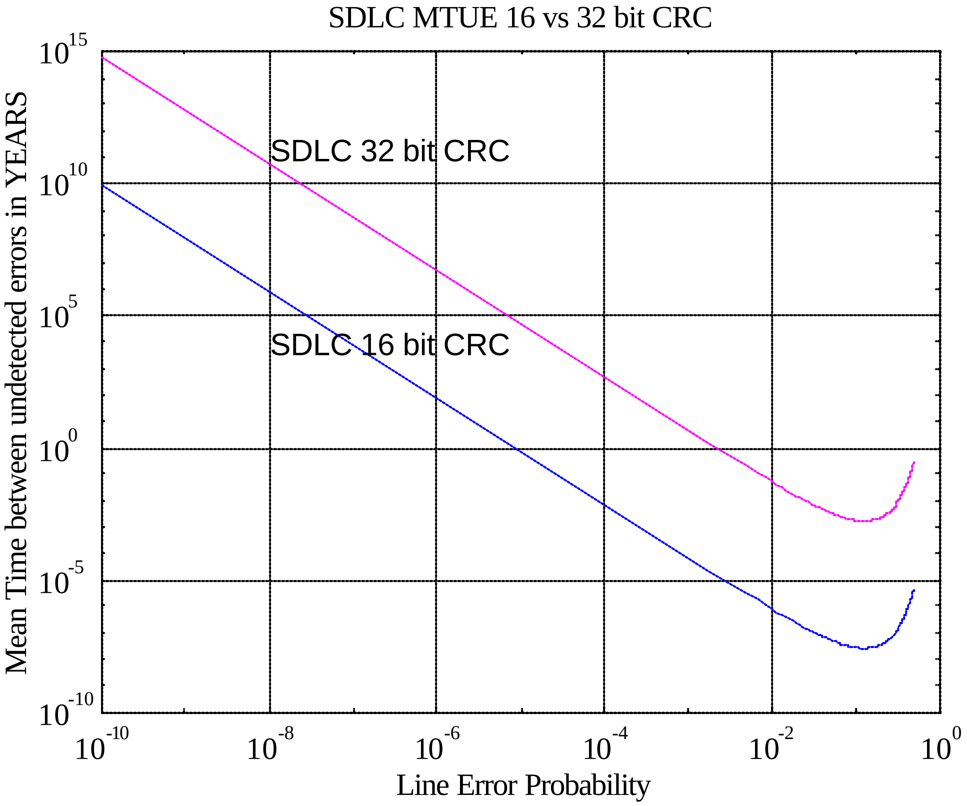

Figure 2: Mean Time between undetected errors: Comparison between 16 and 32 bit CRC.

As the line probability of error varies widely in optical wireless links, due to possible misalignment of the transceivers, or other reasons, such as sudden increase in ambient noise, we observe that the probability of undetected errors also increases as shown above. Choosing 32 bit CRC is better than 16 bit CRC, and increases the mean time between undetected errors by some 10^5 years. The advantages of a 32 bit CRC must be counterweighted by the increase in cost and complexity of the system. However, from Figure 2 the mean time between undetected errors for 16 bit CRC is quite poor even for error rates of the order of 10^{-6}, rendering the link unworkable for high error rates. A 32 bit CRC may be a essential to reduce the undetected errors.

Since we have arbitrarily chosen F=8544 bits for the calculations so far, it is desirable to examine the dependence of MTUE on frame size, due to undetected errors causing valid EOF. This is useful since we have no control on the minimum data size transmitted. Figure 3 shows the variation of the MTUE as a function of frame size. The MTUE varies by approximately 100 years, between small and large frames. The figure shows an improvement in MTUE for smaller frames. This is due to the factor $\frac{p^2}{2}(F-1)$, favouring large frames.

The variation is not so significant and it is found to be the same for other probabilities of line code error, p. The variation of MTUE is much more strongly dependent on ‘p’, as shown in Figure 2.

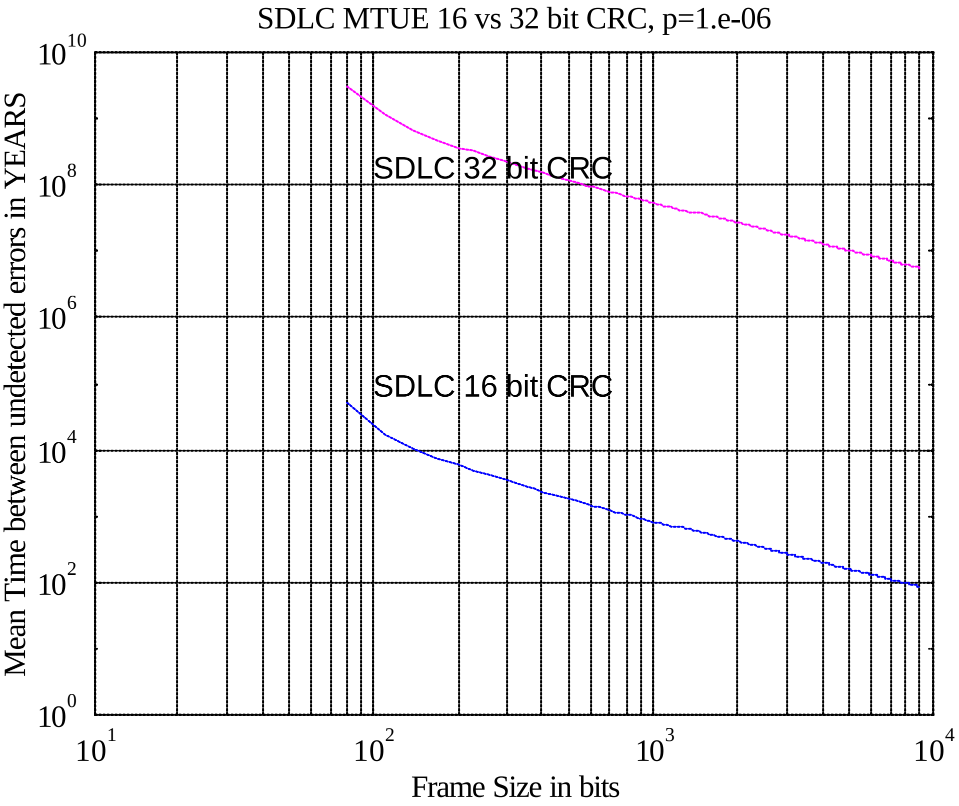

Figure 3: Mean Time between undetected errors dependence on frame size, for fixed p=1.e-06.

5. CONCLUSION

For Manchester line coding and SDLC data framing, an approximate analysis is presented which allows designers of free space optical links to estimate the weakness of their system with the probability of undetected errors as a measure. It has been shown that only two successive errors which invert data 1 into 0 or 0 into 1 can escape the Manchester line code violation 'filter', and following that, the CRC test is the next 'filter' for the undetected errors. A 32 bit CRC improves the mean time between undetected errors by some 10^5 years, over a 16 bit CRC. The weakest part of the SDLC framing/Manchester encoding combination is undetected errors in EOF.

The variation of MTUE is less dependent on the frame size, and varies by approximately 100 years between small and large data frames, largely regardless of the probability of line code error rate value.

6. REFERENCES:

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