Analysis of Leaky Modes and Bragg Fibers Using Transmission Line Equivalent T-Circuits

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***Abstract*—In this letter, we have extended the transmission line (TL) technique to calculate the effective index of leaky modes and demonstrated it by using the model of Bragg fibers. From Maxwell's equations we derived a TL equivalent circuit for the Bragg fiber and we demonstrate how it can be used to determine the mode effective index. Transverse magnetic modes of Bragg fibers and mode bandgap structures can be analyzed using the TL method. Furthermore, we demonstrate that this model allows us to carry out the inverse problem and synthesize the exact refractive index profile if the desired Bragg fiber near field is available. The accuracy of the reconstructed waveguides is also examined numerically.**

***Index Terms*—Bragg fibers, photonic bandgap.**

I. Introduction

RECENTLY, all-dielectric waveguides have been introduced that confine optical light by means of one-dimensional, two-dimensional, and three-dimensional photonic bandgaps [1]. These new designs have the potential advantage that light propagates mainly through the empty core of a hollow waveguide, thus minimizing effects associated with material nonlinearities and absorption losses.

The Bragg fiber combines some of the best features of the metallic coaxial cable and the dielectric waveguides. Several numerical approaches have been used to analyze the modal properties of Bragg fibers with air core and periodic coaxial claddings. In [2], Bragg fibers were successfully analyzed using the transfer matrix method, where the Bragg modes were considered as quasi-modes with minimum radiation loss. In [3], the photonic bandgap concept was used in the transfer matrix method. It obtained the bandgap by searching for the fast increasing solutions. The increasing numerical errors make the field calculation very sensitive to the propagation constants. In [4], periodic alternate layers were approximated by planar Bragg stacks using asymptotic approximation of Bessel functions; therefore, the Bloch theorem can be used to obtain an analytical eigenequation. The drawback is that this simple asymptotic analysis may fail if the Bragg fiber core becomes too small and it is difficult to estimate the accuracy of the asymptotic results. In [5], plane wave expansion for photonic crystal calculation was used along with the supercell concept. Since a Bragg fiber is not strictly a photonic crystal, a plane wave method is not quite suitable or efficient. The multipole method is a potential approach to calculate the modal properties of microstructured optical fibers [6], however, it is at present limited to designs composed of nonintersecting circular inclusions.

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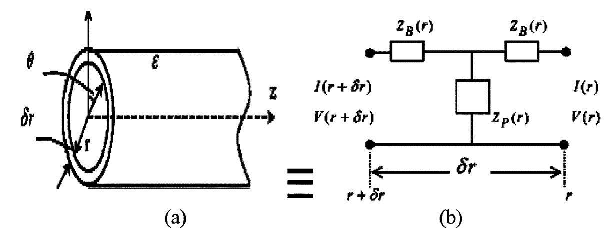

Fig. 1. (a) Homogeneous cylindrical layer. (b) Equivalent circuit of such a layer of a dielectric optical fiber.

Here we extend and apply our transmission line (TL) technique [7], [8] to leaky mode circular waveguides using the Bragg fiber with air core and periodic coaxial claddings as an example where the bandgap structures of transverse magnetic (TM) modes can be obtained.

II. TL Representation of Optical Fibers

We divide an optical fiber into a large number of homogeneous concentric cylindrical layers of thickness δr, permittivity ε, permeability μ, and conductivity σ. Fig. 1(a) and (b) shows the equivalent T-circuit.

The Maxwell's equations for any such layer can be written as

$$\left.\begin{aligned} \beta r E_\theta - lE_Z &= \omega\mu r H_r \\ lH_Z - \beta r H_\theta &= (\omega\varepsilon - j\sigma) r E_r \\ \frac{\partial(\omega\mu r H_r)}{\partial r} &= -j\omega\mu(lH_\theta + \beta r H_Z) \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} \frac{\partial[(\omega\varepsilon - j\sigma) r E_r]}{\partial r} &= -(\sigma + j\omega\varepsilon)(lE_\theta + \beta r E_Z) \\ \frac{\partial(lH_\theta + \beta r H_Z)}{\partial r} &= -\frac{\gamma^2}{j\omega\mu}\omega\mu r H_r + \beta H_Z - \frac{l}{r}H_\theta \\ \frac{\partial(lE_\theta + \beta r E_Z)}{\partial r} &= -\frac{\gamma^2}{\sigma + j\omega\varepsilon}(\omega\varepsilon - j\sigma) r E_r + \beta E_Z - \frac{l}{r}E_\theta \end{aligned}\right\}. \quad (2)$$

We restrict our analysis to the case $\sigma = 0$, $l = 0$ (for transverse electric (TE) and TM modes), $\mu = \mu_0$, $\varepsilon = n^2\varepsilon_0$, where n is the refractive index of the layer at distance r from the axis, l is the wave number of modes propagated in optical waveguides. After algebraic calculation as shown in [7], we obtain the well-known TL equations for TE and TM modes with the solution represented by the T-circuit shown in Fig. 1(b)

$$\left.\begin{aligned} \frac{\partial V_s}{\partial r} = \frac{-\gamma_s^2}{j\omega\varepsilon_0 nF} I_s, \ \frac{\partial V_d}{\partial r} = \frac{-\gamma_d^2}{j\omega\varepsilon_0 nF} I_d \\ \frac{\partial I_s}{\partial r} = -j\omega\varepsilon_0 nFV_s, \ \frac{\partial I_d}{\partial r} = -j\omega\varepsilon_0 nFV_d \end{aligned}\right\}. \quad (3)$$

The equivalent electric T-circuit normalized impedances are

$$\left.\begin{aligned} \overline{Z}_B &= \frac{1}{2}(\delta\overline{r})^2\overline{\gamma}^2_{\substack{s\\d}}\overline{Z}_P \\ \overline{Z}_P &= \frac{Z_0}{jn\overline{r}\delta\overline{r}\overline{\beta}^2} \end{aligned}\right\} \quad (4)$$

where $\overline{r} = rk_0$, $\delta\overline{r} = \delta rk_0$, $\overline{\beta} = \beta/k_0$, $\overline{\gamma}_{\frac{s}{d}}^2 = \gamma_{\frac{s}{d}}^2/k_0^2 = \overline{\beta}^2 - n^2$ $\overline{Z}_p = Z_p k_0$, $\overline{Z}_B = Z_B k_0$, $F = \beta^2 r$

An optical fiber can then be represented as a cascade of T-circuits connected in tandem. The series is terminated with the characteristic impedance of the medium at the axis ($r = 0$) and the outer cladding ($r = \infty$) of the fiber. Using circuit theory, starting from large r in the cladding, we find Z_{out} the total impedance up to the cladding–core boundary and similarly, Z_{in} the total impedance from $r = 0$ to that boundary. We can easily determine Z_{out} and Z_{in} given by

$$Z_{\substack{\mathrm{in}\\\mathrm{out}}} = Z_B(a \pm 1) + \cfrac{1}{\cfrac{1}{Z_P(a\pm1)} + \cfrac{1}{Z_B(a\pm1)+Z_B(a\pm2)+\cfrac{1}{\ddots \cfrac{1}{Z_{\mathrm{Prev}}+Z_B(n-1)}}}} \tag{5}$$

where the characteristic impedance $Z_{\mathrm{prev}} = 0$ at large r when the positive sign is used or it becomes $Z_{\mathrm{prev}} = \infty$ at $r = 0$ when the negative sign is used, a is the core radius. The total circuit resonates when Z_{in} and Z_{out} are equal and opposite, hence $Z_{\mathrm{total}} = Z_{\mathrm{in}} + Z_{\mathrm{out}} = 0$, at the normalized propagation constant value of any mode. Therefore, we obtain the unknown effective index $\overline{\beta}$ using a root searching method which locates the roots of the total impedance of the T-circuits, $Z_{\mathrm{total}} = 0$. The root searching for locating leaky modes is carried out for $0 < \overline{\beta} < n_0$ in the case of air core waveguides ($n_0 = 1$). Similar to [8], we can derive the TM mode radial electric field $E_{\overline{r}}(\overline{r})$ given by

$$E_{\overline{r}}(\overline{r}) = \frac{Z_0 I_s(\overline{r})\sqrt{n(\overline{r})}}{n^2(\overline{r})\overline{r}}. \tag{6}$$

This is the most interesting and powerful advantage for using this approach in determining mode propagation constants.

For the inverse problem of determining the refractive index profile from knowledge of the near field $E_{\overline{r}}(\overline{r})$, we assume the following boundary condition: At $r = \infty$, $n = n_2$ (silica refractive index), and $Z_{\mathrm{prev}} = 0$. We also assume for the inverse problem to have full knowledge of Bragg fiber effective index $\overline{\beta}$. Hence, we can work out $n(\overline{r})$ as follows:

$$n(\overline{r}) = \left[\frac{I_s(\overline{r})Z_0}{2E_{\overline{r}}(\overline{r})\overline{r}}\right]^{2/3} \tag{7}$$

where $Z_0 = 120\pi$ is the free space impedance. Since we know $E_{\overline{r}}(\overline{r})$ and $I_s(\overline{r})$, hence we can calculate $n(\overline{r})$ for all $\overline{r}$.

III. Characteristics of the Bragg Fiber

In general, a Bragg fiber can be described by seven parameters: $n_0, n_1, n_2, r_0, d_1, d_2$, and N (Fig. 2), where they represent the refractive indexes of the core and the alternating high-low index layers, the core radius, layer thickness, and the number of cladding pairs, respectively. The effect of changing one or more of these parameters has been considered previously [9], but only over a limited range and not in the context of single modedness.

Consider a Bragg fiber having the structure like the one shown in Fig. 2. The core of the waveguide is an air hole, whose radius is denoted by r_0, at the center of the cross section. The cladding consists of a coaxial periodic structure, where the radius of the ith boundary is defined by

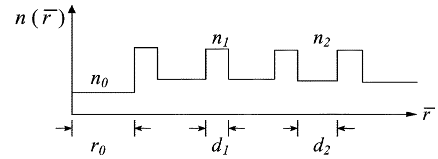

Fig. 2. Refractive index profile of a Bragg fiber, the air-core $n_0 = 1$ and radius r_0, the alternating high-low indexes and thickness $n_,$, d_1 and $n_,$, d_2, respectively.

$$r_i = \begin{cases} n(d_1 + d_2) + r_0, & i = 2n \\ n(d_1 + d_2) + d_1 + r_0, & i = 2n+1 \end{cases} \quad (n=0,1,2,3\ldots). \tag{8}$$

The refractive index for the ith layer is defined by

$$n_i = \begin{cases} n_1, & i = 2n-1 \\ n_2, & i = 2n \end{cases} \quad (n = 1,2,3\ldots). \tag{9}$$

To compare our results to other techniques, we choose the same parameters and results from [4] for convenience, for a Bragg fiber. The cladding structure of the Bragg fiber is: $n_1 = 4.6$ and thickness $d_1 = 0.333d$, $n_2 = 1.6$ and the thickness $d_2 = 0.667d$. Here, $d = d_1 + d_2$ is the unit length of periodicity (cladding pair) of the multilayered structure. The Bragg fiber has a core with $n_0 = 1$ and the radius $r_0 = 1.4d$. In practice, Bragg fibers have a finite number of layers and support only leaky modes [9].

IV. Numerical Results and Discussion

The Bragg fiber leaky mode effective index can be achieved by using the root searching method locating the resonances of the electric T-circuit cascade. The solution to this problem allows working out the bandgap structures of TM modes, for example, and also the near electric field distribution of the Bragg fiber. The accuracy of the effective index is related to the number of concentric homogeneous cylindrical layers where we use 300 layers (300 equivalent T-circuits) with good accuracy. A total of five cladding pairs are used for the Bragg fiber, which should provide good mode confinement based on our experience. Hence, the thickness of the homogeneous cylindrical layers is $\delta\overline{r} = [5(d_1 + d_2) + r_0]/300$. This results in the band diagram of Fig. 3. Only one guided mode (TM_{01}) exists in the bandgap. This implies that in this range only TM_{01} mode can propagate in agreement with [4].

Using (6), the TM_{01} electric field $E_{\overline{r}}(\overline{r})$ is plotted in Fig. 4(a) with parameters effective index $\overline{\beta} = 1.25$ and V-value $V = 8$. For high index difference, boundary matching imposes jump discontinuities at the periodic layer interfaces for $E_{\overline{r}}(\overline{r})$. For the inverse problem, applying the refractive index profile synthesis method we developed in [8], and using (7), together with the electric field $E_{\overline{r}}(\overline{r})$ of the TM_{01}, we can synthesize the original Bragg refractive index profile directly. The result is shown in Fig. 4(b). In order to compare its accuracy with the original refractive index, Fig. 4(c) shows the percent error versus the normalized radius. The error in refractive index shows small oscillations about the exact value in the cladding, and the values in the air core are very accurate. The oscillations in the cladding

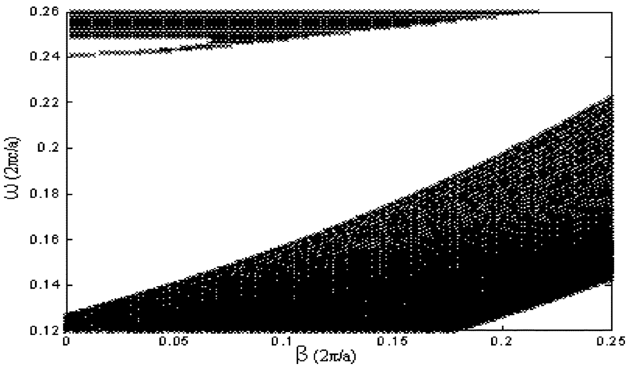

Fig. 3. Band diagram of the Bragg fiber: core index $n_0 = 1$, thickness $r_0 = 1.4d$, layer1 index $n_1 = 4.6$, thickness $d_1 = 0.333d$, layer2 index $n_2 = 1.6$, thickness $d_2 = 0.667d$, $d = d_1 + d_2$ is the unit length of periodicity of the multilayered structure.

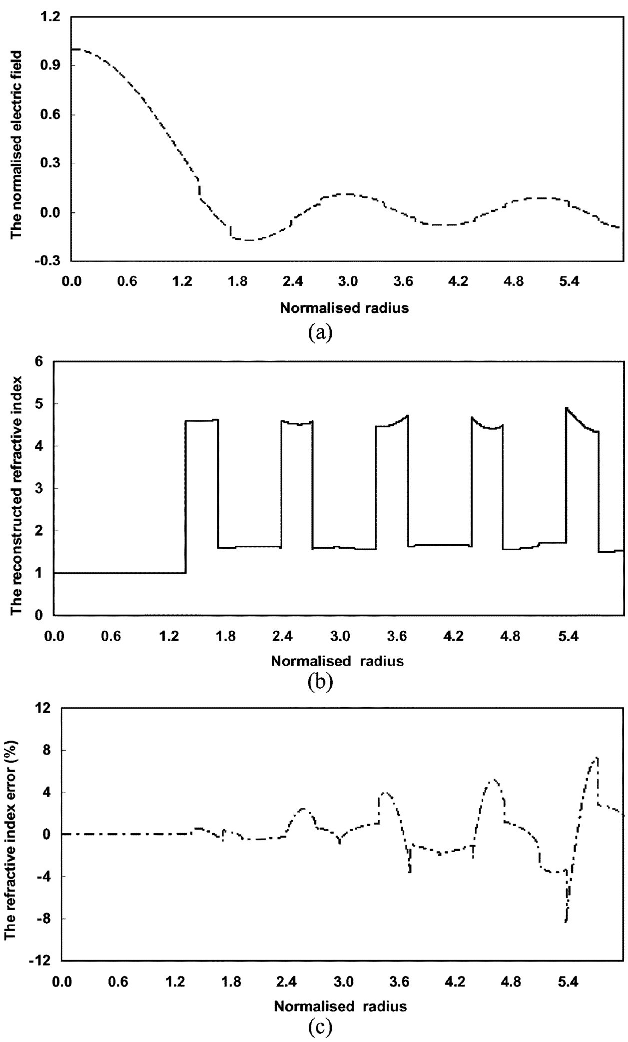

Fig. 4. (a) Electric field profile for the TM_{01} mode Bragg fiber. (b) The reconstructed Bragg structure refractive index profile from the near field of the TM_{01} mode. (c) The percent error in the reconstructed refractive index profile, (the difference between the exact and reconstructed refractive index profile).

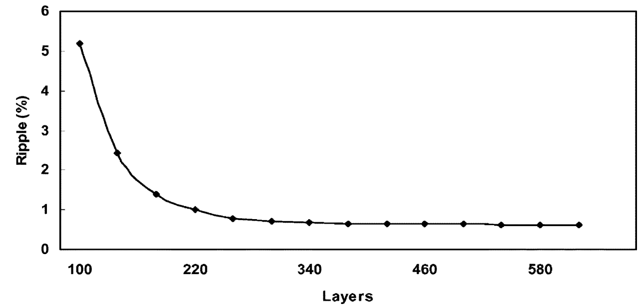

Fig. 5. Refractive index difference, maximum percent ripple (error) of the synthesized refractive index versus number of cylindrical layers.

depend on the number of homogeneous cylindrical layers we use for the reconstruction of the index, as shown in Fig. 5. We can see in Fig. 5 that less than 1% error due to the ripple in Δn can be achieved with 300 up to 500 layers. This could be due to the use of the approximations (4) instead of the exact ones [8, eq. (7)], as well as the fact that the layer must be very thin in order to apply this theory accurately. To save computation time, we use 300 layers in our calculations.

V. Conclusion

We have extended a novel and accurate T-circuit technique to work out the propagation and band diagrams of leaky mode waveguides and specifically Bragg fiber TM modes. We also demonstrated that we can also synthesize the refractive index profile from its mode near field. The method uses TL principles and relies on the modeling of a cascade of thin uniform homogeneous cylindrical layers of a Bragg fiber to TL circuits. Simulation results demonstrate the potential of this new method for modeling properties of Bragg fibers.

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