Optical Fibre Refractive Index Reconstruction from Mode Near Field

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ABSTRACT. A novel and accurate refractive index profile synthesis method for optical fibres is developed using knowledge of the fundamental or higher order mode near field. This fundamental method is based on inverse transmission line principles. From Maxwell's equations, we derive a transmission line equivalent circuit for a circularly symmetric optical fibre with arbitrary refractive index. We demonstrate how to use this model to carry out the inverse problem of synthesis of the refractive index profile from mode near field data. We apply this method to construct waveguides supporting unusual near field patterns, and the accuracy of the reconstructed refractive index profile is examined numerically.

1. Introduction

The refractive index profile of an optical fibre plays an important role in characterizing the properties of the optical fibre. It allows the determination of the fibre's numerical aperture (NA) and of the number of modes propagating within the optical fibre, while defining intermodal and profile dispersion caused by the optical fibre itself. It is essential to establish an efficient and accurate method for measuring the refractive index profile. A number of techniques, [1], [2], [3], have been proposed for determining the refractive index distribution of optical fibres from the propagation mode near field. The most well known rely on the seminal theoretical work by Morishita [2], which relies on an inverse solution of the scalar wave equation for the refractive index profile. In [1], the measurement of the near field intensity is improved using a scanning optical microscopy technique rather than conventional optics. Improvements from [2], have been recently reported in [3], which is a robust method to noise and errors, and non-iterative, but reported for planar waveguides only. We have shown that our transmission line technique can be applied in optical fibres and can determine exactly the mode propagation constants [4]-[5], and cutoff wavelengths of waveguide modes [6]. In general from knowledge of the monomode optical fibre near electric field, we can even synthesize the exact refractive index profiles numerically using this powerful technique [7].

In this paper, first we are extending our results and demonstrate that the same refractive index profile can be synthesised not only from the mode field of the *HE₁₁* mode but also from knowledge of higher order mode fields, hence confirming the generality of this method. We then proceed with synthesizing refractive index profiles which support unusual mode fields such as linear, or sigmoid. Sigmoid fields are very interesting because they offer a mode fields distribution in the core which is very flat, unlike ordinary step index fibres which support Bessel function mode field distributions.

2. Transmission line Representation of Optical fibres

We divide a cylindrical optical waveguide into a large number of concentric homogeneous cylindrical layers of thickness δr, permittivity ε, permeability μ and conductivity σ in Fig.1.

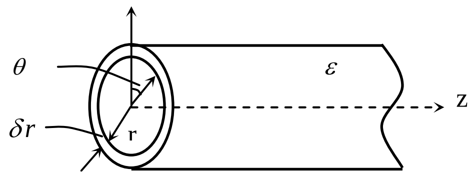

Figure1. Homogeneous optical fiber thin cylindrical layer

The E and H components of Maxwell's equations for any such layer can be written as (1) and (2), [4]

$$\left.\begin{aligned} &\beta r E_\theta - l E_Z = \omega\mu r H_r \\ &l H_Z - \beta r H_\theta = (\omega\varepsilon - j\sigma) r E_r \\ &\frac{\partial(\omega\mu r H_r)}{\partial r} = -j\omega\mu(l H_\theta + \beta r H_Z) \end{aligned}\right\} \quad (1)$$

where $\gamma^2 = \beta^2 + (\frac{l}{r})^2 - \omega^2\mu\varepsilon + j\omega\mu\sigma$, β is the propagation constant, l is the azimuthal mode number (integer), and ω is the mode frequency. We restrict our analysis to the case $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = n^2\varepsilon_0$, where n is the refractive index of the layer at distance r from the axis.

Equation (3) represents two independent transmission lines with voltages V_s, V_d and currents I_s, I_d.

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$$\left.\begin{aligned}
&\frac{\partial[(\omega\varepsilon - j\sigma)rE_r]}{\partial r} = -(\sigma + j\omega\varepsilon)(lE_\theta + \beta rE_Z) \\
&\frac{\partial(lH_\theta + \beta rH_Z)}{\partial r} = -\frac{\gamma^2}{j\omega\mu}\omega\mu rH_r + \beta H_Z - \frac{l}{r}H_\theta \\
&\frac{\partial(lE_\theta + \beta\, rE_Z)}{\partial r} = -\frac{\gamma^2}{\sigma + j\omega\varepsilon}(\omega\varepsilon - j\sigma)rE_r \\
&\qquad + \beta\, \mathrm{E}_Z - \frac{l}{r}E_\theta
\end{aligned}\right\} \quad (2)$$

After some algebra similarly to [7], (1) and (2) can be transformed into

$$\left.\begin{aligned}
&\frac{\partial V_s}{\partial r} = \frac{-\gamma_s^2}{j\omega\varepsilon_0 nF}I_s\,, \quad \frac{\partial I_s}{\partial r} = -j\omega\varepsilon_0 nFV_s \\
&\frac{\partial V_d}{\partial r} = \frac{-\gamma_d^2}{j\omega\varepsilon_0 nF}I_d\,, \quad \frac{\partial I_d}{\partial r} = -j\omega\varepsilon_0 nFV_d
\end{aligned}\right\} \quad (3)$$

where $\gamma_{\substack{s\\d}}^2 = \beta^2 + (\frac{l}{r})^2 - n^2k_0^2 \mp \frac{2nk_0\beta l}{(\beta r)^2 + l^2}$

(- for HE and + for EH modes).

The corresponding characteristic impedances are

$$\left. Z_s = \frac{\gamma_s}{j\omega\varepsilon_0 nF}\,, \qquad Z_d = \frac{\gamma_d}{j\omega\varepsilon_0 nF} \right\} \quad (4)$$

The above equations are recognized as the well known transmission line equations with the solution represented by the following electric circuit Fig.2.

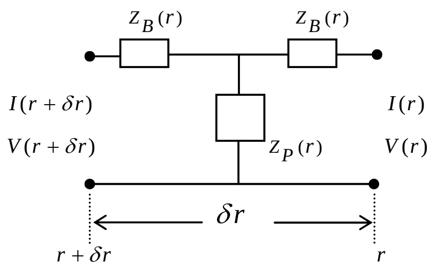


Figure2. Equivalent circuit of a dielectric waveguide

With impedances

$$\left.\begin{aligned}
&Z_B = Z_{\substack{s\\d}} \tanh(\gamma_{\substack{s\\d}} \frac{\delta r}{2}) \\
&Z_P = \frac{Z_{\substack{s\\d}}}{\sinh(\gamma_{\substack{s\\d}} \delta r)}
\end{aligned}\right\} \quad (5)$$

where δr is the length of the transmission line.

Since δr is infinitesimal, $\frac{\delta r}{r}$ <<1, we finally have

$$\left.\begin{aligned}
&Z_B = \frac{1}{2}(\delta r)^2\gamma_{\substack{s\\d}}^2 Z_P \\
&Z_P = \frac{Z_0}{jnr\delta rk_0(\beta^2 + (\frac{l}{r})^2)}
\end{aligned}\right\} \quad (6)$$

Normalizing (6) with respect to k_0 gives

$$\left.\begin{aligned}
&\overline{Z}_B = \frac{1}{2}(\delta\overline{r})^2\overline{\gamma}_{\substack{s\\d}}^2 \overline{Z}_P \\
&\overline{Z}_P = \frac{Z_0}{jn\overline{r}\delta\overline{r}(\overline{\beta}^2 + (\frac{l}{\overline{r}})^2)}
\end{aligned}\right\} \quad (7)$$

where $\overline{r} = rk_0,\ \delta\overline{r} = \delta rk_0,\ \overline{\beta} = \frac{\beta}{k_0}$

$\overline{\gamma}_{\substack{s\\d}}^2 = \gamma_{\substack{s\\d}}^2 / k_0^2 = \overline{\beta}^2 + (\frac{l}{\overline{r}})^2 - n^2 \mp \frac{2n\overline{\beta}l}{(\overline{\beta}\overline{r})^2 + l^2}$

$\overline{Z}_p = Z_p \times k_0,\ \overline{Z}_B = Z_B \times k_0,\ Z_0 = 120\pi$ is the free space impedance.

A waveguide can therefore be modelled as a cascade of T circuits, the impedances, voltages and currents of which depend on the waveguide physical and optical properties. For determining the refractive index profile from knowledge of E_r, we assume the following boundary condition: At $r = \infty$, we assume $Z_{prev} = 0$ and $n=n_2$ (silica refractive index). The equivalent circuit for a cylindrical thin layer, Fig.1, of constant refractive index n and thickness $\delta\overline{r}$ at distance $\overline{r}$ from the core is represented as an electric circuit in Fig.3. From circuit theory, we may use the following recursive relation to determine the values of $Z_{p,n}$, $Z_{B,n}$ and $Z_{prev} = Z_n$.

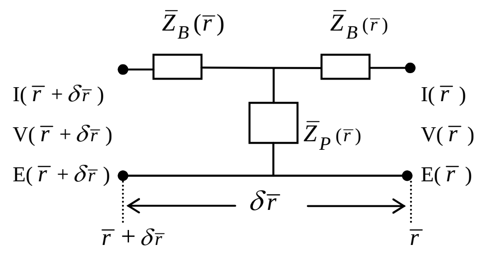


Figure3. Equivalent circuit for a cylindrical thin layer at constant refractive index n and thickness $\delta\overline{r}$ at distance $\overline{r}$ from the core, where V, I ,E are the electric voltages and currents in the circuit and E is the corresponding mode Electric field.

From (8), since we know that $\overline{\beta}$ is the effective refractive index and for typical waveguides lies between n_1 and n_2. For the time being, we also assume we have full knowledge of $\bar{\beta}$. We also know λ_0, $\bar{r}$, $\delta\bar{r}$, $n(\infty) = n_2$.

$$\left.\begin{aligned} \bar{Z}_n &= \frac{(\bar{Z}_{n-1} + \bar{Z}_{B,n})\bar{Z}_{P,n}}{\bar{Z}_{n-1} + \bar{Z}_{B,n} + \bar{Z}_{P,n}} + \bar{Z}_{B,n} \\ \bar{Z}_{B,n} &= \frac{1}{2}\bar{\gamma}^2 \delta\bar{r}^2 \bar{Z}_P \\ \bar{Z}_{P,n} &= \frac{Z_0}{n(r)\bar{r}\delta\bar{r}(\bar{\beta}^2 + \frac{l^2}{\bar{r}^2})} \end{aligned}\right\} \quad (8)$$

Where $\bar{\gamma}^2 = \bar{\beta}^2 + \frac{l^2}{\bar{r}^2} - n^2(r) - \frac{2\bar{\beta} l n(r)}{(\bar{\beta}^2\bar{r}^2 + l^2)}$

Hence $I(\bar{r} + \delta\bar{r}), V(\bar{r} + \delta\bar{r})$ can be calculated for any radius

$$E(\bar{r}) = \frac{I_E(\bar{r})Z_0}{n^2\bar{r}}, \quad I_s = \frac{2I_E}{\sqrt{n}} \quad (9)$$

Since we know $E(\bar{r})$ and $I_s(\bar{r})$, hence we can calculate $n(\bar{r})$, and synthesize the refractive index profile..

$$n(\bar{r}) = \left[\frac{I_s(\bar{r})Z_0}{2E(\bar{r})\bar{r}}\right]^{2/3} \quad (10)$$

3. Numerical Results

Fig.4 shows the exact calculated fields of HE_{11}, HE_{12} and HE_{21} modes of a segmented core optical fibre. By using (10) with wave number l=1 for HE_{11}, HE_{12} and l=2 for HE_{21}, we can use the inverse method to reconstruct refractive index profile of the segmented optical fibre. Fig.5(a) shows the reconstructed refractive index profile of a segmented optical fibre which agrees with the original refractive indices of n_1=1.51508, n_2=1.508, and n_3 =1.512 using the HE_{11} mode near field data. We compare its accuracy with the original (exact) refractive index in Fig.5(b), which shows the error (%) versus the normalised radius. The error in refractive index shows small oscillations about the exact value in the core. The error in the cladding is much smaller. In Fig.6 we show the effect of inaccuracies in $\bar{\beta}$ on the ripple in the reconstructed Δn for HE_{11}, HE_{12} and HE_{21} modes. We observe that for each mode the ripple increases with using the incorrect $\bar{\beta}$. Since the min. error occurs at the exact $\bar{\beta}$, this gives us a means of locating the unknown $\bar{\beta}$, since we can simply start the reconstruction with $\bar{\beta} = n_2$ and repeat the reconstruction process with a new $\bar{\beta}$ within $n_2 \le \bar{\beta} \le n_1$ until the observed ripple is minimised. At the observed minimum error ripple corresponding to the exact $\bar{\beta}$, the reconstructed refracted index is also exact. HE_{11}, HE_{12} and HE_{21} modes have their own exact $\bar{\beta}$, the values as shown in Fig. 6. The HE_{11} mode has the maximum $\bar{\beta}$ among the three modes.

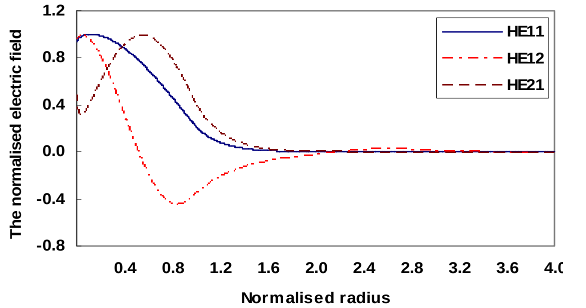
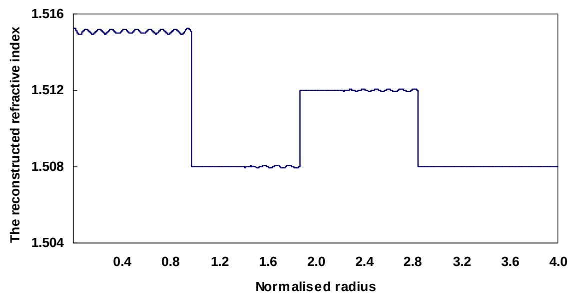
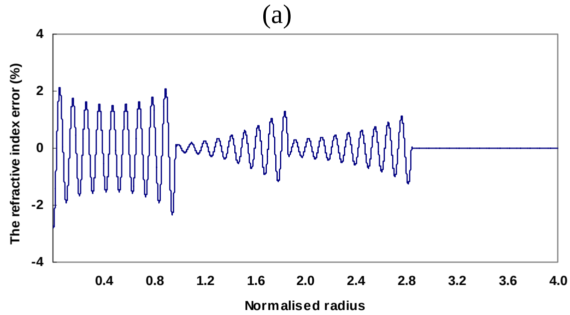


Figure4. The HE_{11} ,HE_{12} and HE_{21} modes near electric fields of a segmented core fibre with n_1=1.51508, n_2=1.508, and n_3 =1.512 and $\lambda = 0.87054\ \mu m$.



(a)



(b)

Figure5. (a).The reconstructed refractive index using (10), and HE_{11} mode field data. (b).The % error in the reconstructed refractive index profile, (the difference between the exact and reconstructed refractive index profile) $(100(n_{exact} - n_{reconstructed})/n_{exact})\%$

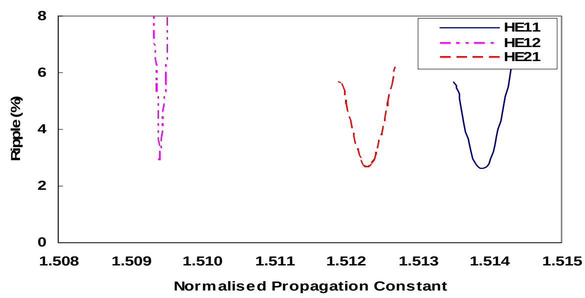
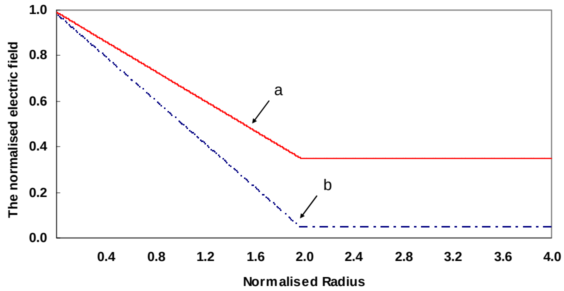
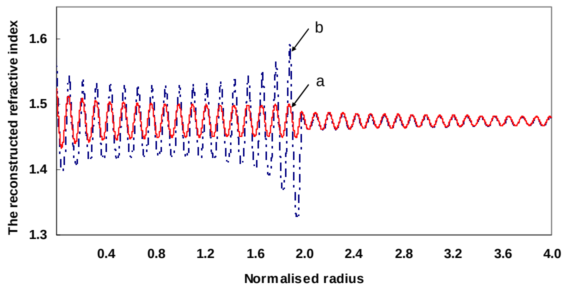


Figure6. The refractive index difference, ripple (%) of the synthesized refractive index versus values for $\bar{\beta}$ offset from exact

In order to illustrate further the power of our technique, let us now consider optical fibres with special near field profile. Fig.7(a) shows two examples of desired triangular, (linear) near electric field with different slopes. In Fig.7(b), we can see the refractive index reconstruction supporting the fields. We demonstarte that our inverse transmission line method reconstructs the refractive index profile successfully for this special near field profile. Fig.(8) is anohter example this time of a sigmoid near field profile. Sigmoids show near constant core field intensities. The fibre refractive index reconstruction using sigmoid near field data is demonstrated in Fig.8(b). In this case, we have forced the cladding index to be constant. This is allowed with our technique and results in deeper gratings in the core since the technique is compensating for the enforcement in a shorter radius range (core only).

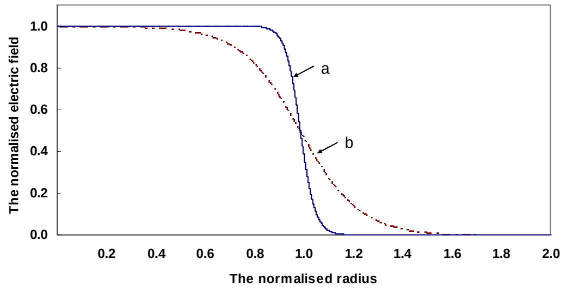


(a)

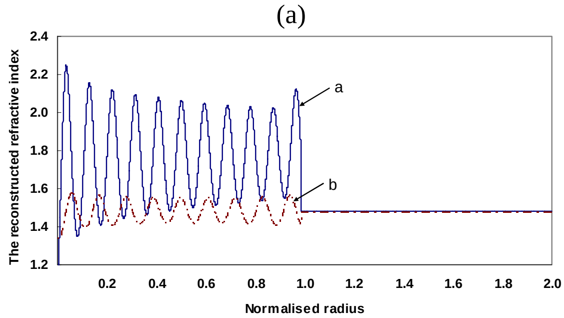


(b)

Figure7.(a).Examples of near electric field (triangular profile) with different slopes. (b).The reconstructed refractive index profiles from the triangular profile electric fields.



(a)



(b)

Figure8.(a).Examples of near electric field (sigmoid profile) with different slopes. (b).The reconstructed refractive index profiles from the sigmoid profile electric fields.

4. Conclusions

In this paper, a new, simple and accurate numerical index profile synthesis procedure has been developed and demonstrated with practical examples. The method uses inverse transmission line principles and relies on the modelling of a thin uniform cylindrical layer of an optical fibre to a transmission line circuit. The method requires knowledge of the near electric field of the optical fibre and the reconstruction is theoretically exact. It allows us to create waveguides supporting modes of nearly arbitrary field distribution, which can be useful in many applications in fibre optic sensing and other applications such as refractive index profiling instrumentation.

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